Definite Integration

Question1

$$\lim_{n \to \infty} \frac{1}{n^2}\left[e^{1/n} + 2e^{2/n} + 3e^{3/n} + \ldots + 2ne^2\right] =$$

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Options:

A.

$e^2 - 1$

B.

$e^2 + 1$

C.

$2e^2 - 2$

D.

$2e^2 + 1$

Answer: B

Solution:

We have,

$$\lim_{n \to \infty} \frac{1}{n^2}\left[e^{1/n} + 2e^{2/n} + 3e^{3/n} + \ldots + 2ne^2\right]$$

$$= \lim_{n \to \infty} \frac{1}{n^2}\left[\sum_{k=1}^{2n} ke^{k/n}\right]$$

$$= \lim_{n \to \infty} \frac{1}{n} \left[\sum_{k=1}^{2n} \left(\frac{k}{n} \right) e^{k/n} \right]$$

$$= \int_0^2 x e^x dx = (x e^x - e^x)_0^2$$

$$= (2e^2 - e^2) - (0 - 1) = e^2 + 1$$

Question2

Let m, n, p, q be four positive integers. If

$$\int_0^{2\pi} \sin^m x \cos^n x dx = 4 \int_0^{\pi/2} \sin^m x \cos^n x dx \int_0^{2\pi} \sin^p x \cos^n x dx = 0$$

$$\int_0^{\pi} \sin^p x \cos^q x dx = 0, a = m + n + p \text{ and } b = m + n + q, \text{ then}$$

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Options:

A.

a is even number and b is odd number

B.

a is odd number and b is even number

C.

Both a and b are even numbers

D.

Both a and b are odd numbers

Answer: D

Solution:

Given, m, n, p, q be four positive inegers

$$\int_0^{2\pi} \sin^m x \cos^n x dx = 4 \int_0^{\pi/2} \sin^m x \cos^n x dx \qquad \ldots (i)$$

$$\int_0^{2\pi} \sin^p x \cos^n x dx = 0 \qquad \ldots(ii)$$

$$\text{And } \int_0^{2\pi} \sin^p x \cos^q x dx = 0 \qquad \ldots(iii)$$

$$a = m + n + p, b = m + n + q$$

From Eq. (i), we know that

$\int_0^{2\pi} f(x) dx = 4 \int_0^{\pi/2} f(x)$

If f is periodic with π and symmetric in all four quadrants

$\Rightarrow \quad m$ and n are even

$\int_0^{2\pi} \sin^p x \cos^n x dx = 0$

$\Rightarrow$ Integrand is odd over symmetric interval $[0, 2\pi]$

Since, n is even so, p is odd also,

$$\int_0^{\pi} \sin^p x \cos^q x dx = 0$$

$$\Rightarrow \quad \sin^p x \cos^q x \text{ is odd about } x = \frac{\pi}{2}$$

$$\Rightarrow \quad q \text{ is odd}$$

$$\therefore a = m + n + p$$

$$= \text{even} + \text{even} + \text{odd} = \text{odd}$$

$$\text{And } b = m + n + p$$

$$= \text{even} + \text{even} + \text{odd} = \text{odd}$$

Question3

$\int_0^2 \sqrt{(x+3)(2-x)} dx =$

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Options:

A.

$\frac{25}{8} \cos^{-1}\left(\frac{1}{5}\right) - \frac{\sqrt{6}}{4}$

B.

$\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)-\frac{\sqrt{6}}{4}$

C.

$\frac{\pi}{2}$

D.

π

Answer: A

Solution:

$I=\int_0^2\sqrt{(x+3)(2-x)}dx$

$$\Rightarrow \quad \int_0^2\sqrt{-x^2-x+6}dx$$
$$\Rightarrow \quad \int_0^2\sqrt{-\left(x^2+x-6\right)}dx$$
$$\Rightarrow \quad \int_0^2\sqrt{\frac{25}{4}-\left(x+\frac{1}{2}\right)^2}dx$$

Put $u=x+\frac{1}{2}\Rightarrow du=dx$

When $x=0, u=\frac{1}{2}$ and $x=2$

$$u=2+\frac{1}{2}=\frac{5}{2}$$
$$I=\int_{\frac{1}{2}}^{\frac{5}{2}}\sqrt{\left(\frac{5}{2}\right)^2-u^2du}$$

$$\Rightarrow \quad \left[\frac{u}{2}\sqrt{\frac{25}{4}-u^2}+\frac{\frac{25}{4}}{2}\sin^{-1}\left(\frac{u}{\frac{5}{2}}\right)\right]_{\frac{1}{2}}^{\frac{5}{2}}$$
$$\Rightarrow \quad \left[\frac{u}{2}\sqrt{\frac{25}{4}-u^2}+\frac{25}{8}\sin^{-1}\left(\frac{2u}{5}\right)\right]_{\frac{1}{2}}^{\frac{5}{2}}$$
$$\Rightarrow \quad \left\{\frac{\frac{5}{2}}{2}\sqrt{\frac{25}{4}-\frac{25}{4}}+\frac{25}{8}\sin^{-1}(1)\right\}-\left\{\frac{1}{4}\sqrt{\frac{25}{4}-\frac{1}{4}}+\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)\right\}$$

$$\Rightarrow \quad 0+\frac{25}{8}\cdot\frac{\pi}{2}-\frac{1}{4}\sqrt{6}-\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)$$
$$\Rightarrow \quad \frac{25\pi}{16}-\frac{\sqrt{6}}{4}-\frac{25}{8}\sin^{-1}\left(\frac{1}{5}\right)$$
$$\Rightarrow \quad \frac{25}{8}\left(\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{5}\right)\right)-\frac{\sqrt{6}}{4}$$
$$\Rightarrow \quad \frac{25}{8}\cos^{-1}\left(\frac{1}{5}\right)-\frac{\sqrt{6}}{4}$$

Question4

$\int_0^{\pi/4} x^2 \sin 2x dx$

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Options:

A.

$\frac{\pi^2-2}{8}$

B.

$\frac{\pi(\pi-2)}{8}$

C.

$\frac{\pi-2}{8}$

D.

$\frac{\pi+2}{8}$

Answer: C

Solution:

$\int_0^{\frac{\pi}{4}} x^2 \sin 2x dx, I=\int x^2 \sin 2x dx$

Put $t=2x \Rightarrow dt=2\cdot dx$

So, $I=\int \frac{t^2 \sin t}{8} dt=\frac{1}{8}\int t^2 \sin t dt$

$$= \frac{1}{8}\left[t^2 \cdot (-\cos t) - 1 \cdot (-2) \cdot \int t \cos t dt\right]$$
$$= \frac{1}{8}\left[-t^2 \cos t + 2 \int t \cdot \cos t dt\right]$$
$$= \frac{1}{8}\left[-t^2 \cos t + 2(t \cdot \sin t - (-\cos t))\right]$$
$$= \frac{1}{8}\left[-t^2 \cos t + 2t \sin t + 2 \cos t\right]$$

$$= \frac{1}{8}\left[(2x)^2 \cos(2x) + 2(2x \cdot \sin(2x)) + 2 \cos(2x)\right]$$
$$= \frac{-x^2 \cos(2x) + x \sin(2x)}{2} + \frac{\cos(2x)}{4}$$

So, $\int_0^{\frac{\pi}{4}} x^2 \sin 2x dx$

$$= \left[\frac{-x^2 \cos(2x) + x \sin(2x)}{2} + \frac{\cos(2x)}{4}\right]_0^{\frac{\pi}{4}}$$
$$= \left\{\frac{-\left(\frac{\pi}{4}\right)^2 \cos\left(2 \times \frac{\pi}{4}\right) + \frac{\pi}{4} \sin\left(2 \times \frac{\pi}{4}\right)}{2} + \frac{\cos\left(\frac{2\pi}{4}\right)}{4}\right\} - \left\{\frac{0}{2} + \frac{\cos(0)}{4}\right\}$$
$$\Rightarrow \left(0 + \frac{\pi}{8} \cdot 1 + \frac{1}{4} \cdot 0\right) - \left(0 + \frac{1}{4}\right)$$
$$= \frac{\pi}{8} - \frac{1}{4} = \frac{\pi - 2}{8}$$

Question5

$\int_{-2\pi}^{2\pi} \sin^4 x \cos^6 x dx =$

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Options:

A.

$\frac{3\pi}{128}$

B.

$\frac{9\pi}{32}$

C.

$\frac{9\pi}{64}$

D.

$\frac{3\pi}{64}$

Answer: D

Solution:

$$\int_{-2\pi}^{2\pi} \sin^4 x \cos^6 x dx$$
$$f(x) = \sin^4 x \cos^6 x$$

Since, $\sin x$ is an odd function and $\sin^2 x$ is an even function and $\cos^6 x$ is an even function.

So, $f(x)$ is an even function.

$$\therefore \int_{-2\pi}^{2\pi} \sin^4 x \cos^6 x dx = 2\int_0^{2\pi} \sin^4 x \cos^6 x dx$$
$$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx\right]$$
$$= 2 \times 2\int_0^{\pi} \sin^4 x \cos^6 x dx$$
$$\left[\because \int_0^{n\pi} f(x)dx = n\int_0^T f(x)dx\right]$$
$$= 4\int_0^{\pi} \sin^4 x \cos^6 x dx$$
$$= 4 \times 2\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x dx$$
$$\left[\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx\right]$$

$$= 8\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x dx$$

[$\because$ If m and n are both even, then

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx =$$

$$\frac{[(m-1)(m-3)\ldots 2 \text{ or } 1][(n-1)(n-3)\ldots 2 \text{ or } 1]}{[(m+n)(m+n-2)\ldots 2 \text{ or } 1]} \times \frac{\pi}{2}]$$

$$= 8 \times \frac{3 \times 1 \times 5 \times 3 \times 1}{10 \times 8 \times 6 \times 4 \times 2} \times \frac{\pi}{2}$$
$$= \frac{3\pi}{64}$$

Question6

$$\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{dx}{\sec^2 x + \left(\tan^{2024} x - 1\right)\left(\sec^2 x - 1\right)} =$$

Options:

A. $\frac{\pi}{20}$

B. $\frac{2\pi}{5}$

C. $\frac{3\pi}{20}$

D. $\frac{3\pi}{5}$

Answer: A

Solution:

$$\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} \frac{dx}{\sec^2 x+\left(\tan^{2024} x-1\right)\left(\sec^2 x-1\right)}$$

$$\int_{\frac{2\pi}{10}}^{\frac{3\pi}{0}} \frac{dx}{1+\tan^2 x+\tan^{2024} x-\tan^2 x}$$

$$=\int_{\frac{2\pi}{10}}^{\frac{3\pi}{10}} \frac{dx}{1+\tan^{2024} x}$$

$$I=\int_{\frac{2\pi}{10}}^{\frac{3\pi}{0}} \frac{\cos^{2024} x}{\sin^{2024} x+\cos^{2024} x} dx$$

Applyh P_IV

$$I=\int_{\frac{\pi}{10}}^{\frac{3\pi}{10}} \frac{\sin^{2024} x}{\cos^{2024} x+\sin^{2024} x} dx$$

On adding Eqs. (i) and (ii), we get

$$2I=\int_{\frac{\pi}{5}}^{\frac{3\pi}{10}} dx=(x)_{\pi/5}^{3\pi/10}=\frac{3\pi}{10}-\frac{2\pi}{10}=\frac{\pi}{10}$$

$$I=\frac{\pi}{20}$$

Question7

$$\int_{-\pi/15}^{\pi/5} \frac{\cos 5x}{1+e^{5x}} dx=$$

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Options:

A. $\frac{1}{5}$

B. $\frac{\sqrt{3}}{10}$

C. $\frac{1}{15}$

D. $\frac{1}{10}$

Answer: B

Solution:

Apply P - IV

$$I=\int_{\frac{-\pi}{15}}^{\frac{\pi}{15}} \frac{\cos(-5x)}{1+e^{-5x}} dx$$

$$=\int_{\frac{-\pi}{15}}^{\frac{\pi}{15}} \frac{e^{5x}\cos 5x}{e^{5x}+1} dx$$

On adding Eqs. (i) and (ii), we get

$2I=\int_{\frac{-\pi}{15}}^{\frac{\pi}{15}} \frac{\cos 5x\left(e^{5x}+1\right)}{e^{5x}+1} dx=\int_{\frac{-\pi}{15}}^{\frac{\pi}{15}} \cos 5x dx$

$=2\int_0^{\frac{\pi}{15}} \cos 5x dx$

$2I=2\frac{1}{5}[\sin 5x]_0^{\frac{\pi}{15}}$

$I=\frac{1}{5}\left[\sin\frac{\pi}{3}-0\right]=\frac{1}{5}\times\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{10}$

Question8

$\frac{3}{25}\int_0^{25\pi}\sqrt{|\cos x-\cos^3 x|} dx=$

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Options:

A. 8

B. 4

C. 1

D. 0

Answer: B

Solution:

To solve the given integral problem, we can start by simplifying the expression inside the integral:

$\frac{3}{25}\int_0^{25\pi}\sqrt{|\cos x-\cos^3 x|}\,dx$

We can rewrite this expression as:

$\frac{3}{25}\int_0^{25\pi}\sqrt{|\cos x(1-\cos^2 x)|}\,dx$

Recognizing that $1-\cos^2 x=\sin^2 x$, this becomes:

$\frac{3}{25}\int_0^{25\pi}\sqrt{|\cos x|\sin^2 x\,dx}$

We further simplify this to:

$\frac{3}{25}\int_0^{25\pi}|\sin x|\sqrt{|\cos x|}\,dx$

Next, we decompose the integral over the period from 0 to 25π into parts over smaller intervals where the function symmetry and periodicity can be utilized:

$$=\frac{3}{25}\left[13\int_0^{\frac{\pi}{2}}\sin x\sqrt{\cos x}\,dx+13\int_{\frac{\pi}{2}}^{\pi}\sin x\sqrt{-\cos x}\,dx-12\int_{\pi}^{\frac{3\pi}{2}}\sin x\sqrt{-\cos x}\,dx-12\int_{\frac{3\pi}{2}}^{2\pi}\sin x\sqrt{\cos x}\,dx\right]$$

For each sub-interval, we compute the integral:

For $\int_0^{\frac{\pi}{2}}$, use the substitution $\cos x$ from 1 to 0.

For $\int_{\frac{\pi}{2}}^{\pi}$, and similarly mapped intervals $\int_{\pi}^{\frac{3\pi}{2}}$, $\int_{\frac{3\pi}{2}}^{2\pi}$, follow similar transformations respecting their respective limits to evaluate each segment.

The integrals resolve as follows:

$$=\frac{3}{25}\left[-13\left[\frac{2(\cos x)^{\frac{3}{2}}}{3}\right]_0^{\frac{\pi}{2}}+13\left[\frac{2(-\cos x)^{\frac{3}{2}}}{3}\right]_{\frac{\pi}{2}}^{\pi}-12\left[\frac{2(-\cos x)^{\frac{3}{2}}}{3}\right]_{\pi}^{\frac{3\pi}{2}}+12\left[\frac{2}{3}(\cos x)^{\frac{3}{2}}\right]_{\frac{3\pi}{2}}^{2\pi}\right]$$

Evaluating each integral gives:

$=\frac{3}{25}\times\frac{1}{3}[-26(0-1)+26(1-0)-24(0-1)+24(1-0)]$

This simplifies to:

$=\frac{1}{25}[26+26+48]=\frac{1}{25}\times 100=4$

Thus, the final calculated value of the integral is 4.

Question9

If m, l, r, s, n are integers such that $9 > m > l > s > n > r > 2$ and $\int_{-2\pi}^{2\pi} \sin^m x \cos^n x dx = 4\int_0^{\pi} \sin^m x \cos^n x dx, \int_{-\pi}^{\pi} \sin^r x \cos^s x dx$ $= 4\int_0^{\pi/2} \sin^r x \cos^s x dx$ and $\int_{-\pi/2}^{\pi/2} \sin^l x \cos^m x dx = 0$, then

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Options:

A. $(s-2)(1-2) = mr$

B. $(s-2)(l+2) = rm + 5$

C. $(s-2)(s+2) = \ln -3$

D. $(I-2)(I+2) = ms - 5$

Answer: C

Solution:

To analyze the integrals, we start by considering the expression $I = \int_{-2\pi}^{2\pi} \sin^m x \cos^n x\, dx$.

Since the given condition is:

$I = 4\int_0^{\pi} \sin^m x \cos^n x\, dx$

We know that:

$\int_{-a}^{a} f(x)\, dx = 2\int_0^{a} f(x)\, dx$

if $f(x)$ is an even function. This suggests that $\sin^m x \cos^n x$ might be even in this case. To satisfy the condition where the integral over the larger symmetric interval is four times the integral over the smaller positive interval, m must be even.

Similarly, consider:

$\int_{-\pi}^{\pi} \sin^r x \cos^s x\, dx = 4\int_0^{\pi/2} \sin^r x \cos^s x\, dx$

Here, for a comparable reason, we must have r as even.

Finally, for:

$\int_{-\pi/2}^{\pi/2} \sin^l x \cos^m x\, dx = 0$

the function should be odd because the integral of an odd function over a symmetric interval around zero results in zero. Thus, l must be odd.

With the constraints $9 > m > l > s > n > r > 2$, and that m and r are even while l is odd, a possible assignment of these integers could be:

$m = 8$

$l = 7$

$s = 6$

$n = 5$

$r = 4$

Next, let's test the option that matches this setup:

For option C, $(s-2)(s+2) = ln - 3$:

$(s-2)(s+2) = (6-2)(6+2) = 4 \times 8 = 32$

The expression $ln - 3$ was a typo originally intended to test our guess, but $32 = 32$ checks out, confirming this is our desired setup.

Question10

$\int_0^\pi \left(\sin^3 x + \cos^2 x\right)^2 dx =$

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Options:

A. $\frac{15\pi}{16} + \frac{8}{15}$

B. $\frac{11\pi}{16} + \frac{8}{15}$

C. $\frac{15\pi}{16} + \frac{4}{15}$

D. $\frac{11\pi}{16} + \frac{4}{15}$

Answer: B

Solution:

Given, $I = \int_0^\pi \left(\sin^3 x + \cos^2 x\right)^2 dx$

$I = \int_0^\pi \left[\sin^3(\pi - x) + \cos^2(\pi - x)\right]^2 dx$

$I = \int_0^\pi \left(\sin^3 x + \cos^2 x\right)^2 dx$

$\Rightarrow I = 2\int_0^{\pi/2} \left(\sin^3 x + \cos^2 x\right)^2 dx$

By applying the property,

$I = 2\int_0^{\pi/2}\left(\cos^3 x+\sin^2 x\right)^2 dx$

On adding Eqs. from (i) and (ii), we get

$$2I = 2\left[\int_0^{\pi/2}\left(\left(\sin^3 x+\cos^2 x\right)^2\right.\right.$$

$$\left.+\left(\cos^3 x+\sin^2 x\right)^2\right]dx$$

$I = \int_0^{\pi/2}\sin^6 x+\cos^6 x+\sin^4 x+\cos^4 x$

$+2\sin^3 x\cos^2 x+2\cos^3 x\sin^2 x dx$

$I = \int_0^{\pi/2}\left(\sin^2 x+\cos^2 x\right)\left(\sin^4 x\right.$

$\left.+\cos^4 x-\sin^2 x\cos^2 x\right)$

$+\sin^4 x+\cos^4 x+2\sin^2 x\cos^2$

$x(\sin x+\cos x)dx$

$I = \int_0^{\pi/2}2\left(\sin^4 x+\cos^4 x\right)+\sin^2 x\cos^2$

$x\left(2\sin x+2\cos x^{-1}\right)$

$I = \int_0^{\pi/2}2\left(1-2\sin^2 x\cos^2 x\right)+\sin^2 x\cos^2$

$x(2\sin x+2\cos x-1)dx$

$I = \int_0^{\pi/2}2-5\sin^2 x\cos^2 x+2\sin^3 x\cos^2$

$x+2\sin^2 x\cos^3 x dx$

Let $I = \int_0^{\pi/2}5\sin^2 x\cos^2 x dx$

$= \frac{5}{4}\int_0^{\pi/2}\sin^2 2x dx$

$= \frac{5}{4}\int_0^{\pi/2}\frac{1-\cos 4x}{2}dx$

$= \frac{5}{8}\left(x-\frac{\sin 4x}{4}\right)_0^{\pi/2} = \frac{5}{8}\left[\frac{\pi}{2}\right] = \frac{5\pi}{16}$

$I_2 = \int_0^{\pi/2}2\sin^3 x\cos^2 x dx$

$= \int_0^{\pi/2}\left(1-\cos^2 x\right)\cos^2 x\sin x dx$

Let $\cos x = t$

$-\sin x dx = dt \Rightarrow \sin x dx = -dt$

Also, $x = 0, t = 1$

$x=\pi/2, t=0$

$$I_2=2\int_1^0\left(t^2-1\right)t^2dt=2\int_1^0\left(t^4-t^2\right)dt$$

$$=2\left[\frac{t^5}{5}-\frac{t^3}{3}\right]_1^0=2\left[-\frac{1}{5}+\frac{1}{3}\right]=\frac{4}{15}$$

$$I_3=2I=\int_0^{\pi/2}\sin^2 x\cos^3 x dx$$

$$I_3=2\int_0^{\pi/2}\sin^2 x\left(1-\sin^2 x\right)\cos x dx$$

Let $\sin x=u \Rightarrow \cos x dx=du$

Also, $x=0, u=0 \Rightarrow x=\pi/2, u=1$

$$I_3=2\int_0^1 u^2\left(1-u^2\right)du$$

$$=2\left[\frac{u^3}{3}-\frac{u^5}{5}\right]_0^1=2\left[\frac{1}{3}-\frac{1}{5}\right]=\frac{4}{15}$$

$$I=[2x]_0^{\pi/2}-\frac{5\pi}{16}+\frac{4}{15}+\frac{4}{15}$$

$$=\pi-\frac{5\pi}{16}+\frac{8}{15}=\frac{11\pi}{16}+\frac{8}{15}$$

Question11

$\int_{\frac{-\pi}{8}}^{\frac{\pi}{8}}\frac{\sin^4(4x)}{1+e^{4x}}dx=$

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Options:

A. $\frac{3\pi}{128}$

B. $\frac{3\pi}{256}$

C. $\frac{3\pi}{64}$

D. $\frac{3\pi}{32}$

Answer: D

Solution:

Given, $I=\int_{-\frac{\pi}{8}}^{\pi/8} \frac{\sin^4(4x)}{1+e^{4x}} dx$

$I=\int_0^{\pi/8}\left[\frac{\sin^4(4x)}{1+e^{4x}}+\frac{\sin^4(-4x)}{1+e^{-4x}}\right] dx$

$I=\int_0^{\pi/8} \sin^4(4x)\left[\frac{1}{1+e^{4x}}+\frac{e^{4x}}{e^{4x}+1}\right] dx$

$I=\int_0^{\pi/8} \sin^4 4x dx$

Let $4x=t \Rightarrow 4dx=dt$

If $x=0 \Rightarrow t=0$

$x=\frac{\pi}{8} \Rightarrow t=\frac{\pi}{2}$

$I=\frac{1}{4}\int_0^{\pi/2} \sin^4 t dt$

$I=\frac{1}{4}\int_0^{\pi/2} \sin^4\left(\frac{\pi}{2}-t\right)$

$=\frac{1}{4}\int_0^{\pi/2} \cos^4 t dt$

$I=\frac{1}{4}\int_0^{\pi/2} \sin^4\left(\frac{\pi}{2}-t\right) dt$

$=\frac{1}{4}\int_0^{\pi/2} \cos^4 t dt$

On adding in Eqs (i) and (ii), we get.

$2I=\frac{1}{4}\int_0^{\pi/2}\left(\sin^4 t+\cos^4 t\right) dt$

$I=\frac{1}{8}\int_0^{\pi/2}\left[1-\frac{1}{2}(\sin 2t)^2\right] dt$

$I=\frac{1}{8}\int_0^{\pi/2}\left[1-\frac{1}{4}(1-\cos 4t)\right] dt$

$I=\frac{1}{8}\int_0^{\pi/2}\left[\frac{3}{4}+\frac{1}{4}\cos 4t\right] dt$

$I=\frac{1}{8}\left(\frac{3t}{4}+\frac{\sin 4t}{16}\right)_0^{\pi/2} \Rightarrow I=\frac{1}{8}\left(\frac{3\pi}{8}\right)=\frac{3\pi}{64}$

Question12

$\int_{\frac{-3}{4}}^{\frac{\pi-6}{8}} \log(\sin(4x+3))dx =$

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Options:

A. $-\frac{\pi}{2}\log 2$

B. $-\frac{\pi}{8}\log 2$

C. $-\frac{\pi}{14}\log 2$

D. $-\frac{\pi}{28}\log 2$

Answer: B

Solution:

Let $I = \int_{-\frac{3}{4}}^{\frac{\pi-6}{8}} \log(\sin(4x+3))dx \quad \ldots \text{(i)}$

$$\Rightarrow I = \int_{-\frac{3}{4}}^{\frac{\pi-6}{8}} \log\left(\sin\left(4\left(\frac{\pi-6}{8}+\left(-\frac{3}{4}\right)-x\right)+3\right)\right)dx$$

$$\Rightarrow I = \int_{-\frac{3}{4}}^{\frac{\pi-6}{8}} \log\left(\sin\left(\frac{\pi}{2}-(4x+3)\right)\right)dx$$

$$\Rightarrow I = \int_{-\frac{3}{4}}^{\frac{\pi-6}{8}} \log(\cos(4x+3))dx \quad \ldots \text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-\frac{3}{4}}^{\frac{\pi-6}{8}} \log(\sin(4x+3)\cos(4x+3))dx$$

$$\Rightarrow 2I = \int_{-\frac{3}{4}}^{\frac{\pi-6}{8}} \log\left(\frac{\sin(2(4x+3))}{2}\right)dx$$

$$\Rightarrow 2I = \int_{-\frac{3}{4}}^{\frac{\pi-6}{8}} \log(\sin(2(4x+3)))dx$$

$$-\int_{-\frac{3}{4}}^{\frac{\pi-6}{8}}(\log 2)dx$$

$$\Rightarrow 2I = I - (\log 2)\left[\frac{\pi-6}{8} - \left(-\frac{3}{4}\right)\right]$$

$$\Rightarrow I = -\frac{\pi}{8}\log 2$$

Question13

$\int_0^{16} \frac{\sqrt{x}}{1+\sqrt{x}}dx =$

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Options:

A. $8 + 2\log 2$

B. $8 + \log 2$

C. $8 + 2\log 5$

D. $4 + \log 5$

Answer: C

Solution:

Let $I = \int_0^{16} \frac{\sqrt{x}}{1+\sqrt{x}}dx$

Put $1 + \sqrt{x} = t$

$$\Rightarrow \quad \frac{1}{2\sqrt{x}}dx = dt$$

$\Rightarrow \quad dx = 2(t-1)dt$

When $x = 0, t = 1$

When $x = 16, t = 5$

$\therefore I = \int_1^{5t-1}$

$t(2(t-1))dt$

$= 2\int_1^{5t^2-2t+1}$

$= 2\int_1^5 \left(t - 2 + \frac{1}{t}\right)dt$

$= 2\left[\frac{t^2}{2} - 2t + \log t\right]_1^5$

$= 2\left[\left\{\frac{25}{2} - 10 + \log 5\right\} - \left\{\frac{1}{2} - 2 + \log 1\right\}\right]$

$= 2\left[\frac{5}{2} + \log 5 + \frac{3}{2} + 0\right]$

$= 8 + 2\log 5$

Question14

$\int_0^{32\pi} \sqrt{1 - \cos 4x} dx =$

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Options:

A. $16\sqrt{2}$

B. $32\sqrt{2}$

C. $128\sqrt{2}$

D. $64\sqrt{2}$

Answer: D

Solution:

$$\text{Let } I=\int_0^{32\pi}\sqrt{1-\cos 4x}dx$$

$$=\int_0^{32\pi}\sqrt{1-\left(1-2\sin^2 2x\right)}dx$$

$$=\int_0^{32\pi}\sqrt{2}|\sin 2x|dx$$

The period of $|\sin 2x|$ is $\frac{\pi}{2}$

$$\therefore \quad I=64\int_0^{\pi/2}\sqrt{2}\sin 2xdx$$

$$=64\sqrt{2}\left[\frac{-\cos 2x}{2}\right]_0^{\pi/2}$$

$$=32\sqrt{2}[-\cos\pi+\cos 0]$$

$$=32\sqrt{2}[-(-1)+1]=64\sqrt{2}$$

Question15

If $f(x)=\int\frac{\sin 2x+2\cos x}{4\sin^2 x+5\sin x+1}dx$ and $f(0)=0$, then $f(\pi/6)=$

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Options:

A. $\log\frac{3}{4}$

B. $2\log 2$

C. $\frac{1}{2}\log 3$

D. 1

Answer: C

Solution:

We have,

$$f(x)=\int \frac{\sin 2x+2\cos x}{4\sin^2 x+5\sin x+1}dx$$

$$=\int \frac{2\cos x(\sin x+1)}{(4\sin x+1)(\sin x+1)}dx$$

$$=\int \frac{2\cos x}{4\sin x+1}dx$$

Let $4\sin x+1=t \Rightarrow 4\cos x dx=dt$

$$\Rightarrow f(x)=\frac{1}{2}\int \frac{dt}{t}=\frac{1}{2}\log t+C$$

$$=\frac{1}{2}\log(4\sin x+1)+C$$

$\because f(0)=0 \Rightarrow C=0$

$$\therefore f(x)=\frac{1}{2}\log(4\sin x+1)$$

$$f\left(\frac{\pi}{6}\right)=\frac{1}{2}\log\left(\frac{4}{2}+1\right)=\frac{1}{2}\log 3$$

Question16

$\int_{-2}^{2} x^4\left(4-x^2\right)^{\frac{7}{2}} dx=$

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Options:

A. 4π

B. $\frac{\pi}{16}$

C. 28π

D. $\frac{3\pi}{128}$

Answer: C

Solution:

Let $I=\int_{-2}^{2} x^4\left(4-x^2\right)^{7/2} dx$

$$I=2 \int_0^2 x^4\left(4-x^2\right)^{7/2} dx$$

$[\because f(x)$ is even function $]$

put $x=2 \sin \theta \Rightarrow dx=2 \cos \theta d\theta$

when $x=0, \theta=0$ and $x=2, \theta=\frac{\pi}{2}$

$$\therefore I=2 \int_0^{\pi/2} 2^4 \sin^4 \theta(2 \cos \theta)^7 \cdot 2 \cos \theta d\theta$$

$$I=2 \cdot 2^4 \cdot 2^8 \int_0^{\pi/2} \sin^4 \theta \cos^8 \theta d\theta$$

$I=2^{13}$.

$\frac{(4-1)(4-3)(8-1)(8-3)(8-5)(8-7)}{12 \times 10 \times 8 \times 6 \times 4 \times 2} \times \frac{\pi}{2}$

(by Wall's formulae)

$$=\frac{2^{13} \times 3 \times 1 \times 7 \times 5 \times 3 \times 1 \times \frac{\pi}{2}}{12 \times 10 \times 8 \times 6 \times 4 \times 2}$$

$$=\frac{2^6 \times 3 \times 7 \times 5 \times 3 \times \pi}{6 \times 5 \times 4 \times 3 \times 2}=28\pi$$

Question17

$\int_0^{\pi/4} \frac{\sec x}{1+2 \sin^2 x} dx=$

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Options:

A. $\frac{1}{3}\log(\sqrt{2}+1)+\frac{\pi\sqrt{2}}{12}$

B. $\frac{2}{3}\log(\sqrt{2}+1)+\frac{\pi\sqrt{2}}{6}$

C. $\frac{1}{6}\log(\sqrt{2}-1)+\frac{\pi}{12}$

D. $\frac{1}{4}\log(\sqrt{2}-1)-\frac{\pi\sqrt{3}}{6}$

Answer: A

Solution:

$$\int_0^{\frac{\pi}{4}} \frac{\sec x dx}{1+2\sin^2 x}$$
$$=\int_0^{\pi/4} \frac{\cos x dx}{\cos^2 x\left(1+2\sin^2 x\right)}$$

$$=\int_0^{\pi/4} \frac{\cos x dx}{\left(1-\sin^2 x\right)\left(1+2\sin^2 x\right)}$$
Let $\sin x = t \Rightarrow \cos x dx = dt$
$$=\int_0^{1/\sqrt{2}} \frac{dt}{\left(1-t^2\right)\left(1+2t^2\right)}$$
$$=\int_0^{1/\sqrt{2}} \left(\frac{A}{1-t}+\frac{B}{1+t}+\frac{Ct+D}{1+2t^2}\right)dt$$

$$=\int_0^{\frac{1}{\sqrt{2}}} \frac{1}{6}\left(\frac{1}{1-t}\right)dt+\frac{1}{6}\int_0^{\frac{1}{\sqrt{2}}} \frac{dt}{t+1}+\frac{2}{3}\int_0^{\frac{1}{\sqrt{2}}} \frac{dt}{1+2t^2}$$
$$\left(\because A=\frac{1}{6}=B \text{ and } C=0, D=\frac{2}{3}\right)$$
$$\Rightarrow -\frac{1}{6}[\ln(1-t)]_0^{\frac{1}{\sqrt{2}}}+\frac{1}{6}[\ln(1+t)]_0^{\frac{1}{\sqrt{2}}}$$
$$+\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{2}}\left[\tan^{-1}(\sqrt{2}t)\right]$$
$$=-\frac{1}{6}\ln\left(1-\frac{1}{\sqrt{2}}\right)+\frac{1}{6}\left(\ln\left(1+\frac{1}{\sqrt{2}}\right)\right)+\frac{1}{3\sqrt{2}}\tan^{-1}(1)$$
$$=\frac{1}{3}\ln(\sqrt{2}+1)+\frac{1}{3\sqrt{2}}\cdot\frac{\pi}{4}$$
$$=\frac{1}{3}\ln(\sqrt{2}+1)+\frac{\pi\sqrt{2}}{12}$$

Question18

$$\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+\ldots\ldots+\frac{1}{n}\sec^2 1\right]=$$

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Options:

A. $\frac{1}{2}\sec(1)$

B. $\frac{1}{2}\text{cosec}(1)$

C. $\tan(1)$

D. $\frac{1}{2}\tan(1)$

Answer: D

Solution:

Now,

$$\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+\ldots+\frac{1}{n}\sec^2 1\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\cdot\frac{n}{n}\cdot\sec^2\frac{n^2}{n^2}=\int_0^1 x\sec^2 x^2 dx$$

$$=\frac{1}{2}\int_0^1 2x\sec^2 x^2 dx$$

Let $x^2=z\Rightarrow 2xdx=dz$

$$=\frac{1}{2}\int_0^1\sec^2 zdz=\frac{1}{2}(\tan z)_0^1$$

$$=\frac{1}{2}\tan 1$$

Question19

$$\int_2^5\sqrt{\frac{5-x}{x-2}}dx=$$

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Options:

A. π

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Given, $\int_2^5 \sqrt{\frac{5-x}{x-2}} dx$

Let x

$x-2=z^2 \Rightarrow dx=2zdz$

$$= \int_0^{\sqrt{3}} \frac{\sqrt{5-(z^2+2}}{z} 2zdz$$

$$= 2\int_0^{\sqrt{3}} \sqrt{3-z^2} dz$$

$$= 2\left[\frac{z\sqrt{3-z^2}}{2} = \frac{3}{2}\sin^{-1}\frac{z}{\sqrt{3}}\right]_0^{\sqrt{3}}$$

$$= 2\left[\frac{3}{2}\sin^{-1}(1)\right] - 3\sin^{-1}(1) = \frac{3\pi}{2}$$

Question20

$$\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x dx =$$

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Options:

A. $\frac{\pi}{256}$

B. $\frac{\pi}{512}$

C. $\frac{3\pi}{512}$

D. $\frac{5\pi}{512}$

Answer: C

Solution:

Given, $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x d x$

Using Wallis formula,

$$\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x d x, m, n \in z$$

$$(m-1)(m-3) \ldots 2 \text{ or } \cdot (n-1)$$
$$=\frac{(n-3) \ldots 2 \text{ or } 1}{(m+n)(m+n-2) \ldots 2 \text{ or } 1} \times \lambda$$

where, $\lambda=\frac{\pi}{2}, m$ and n both even.

$$\text{So, Let} \quad \begin{aligned} \mathrm{I} & =\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x d x \\ & =\frac{5 \times 3 \times 1 \times 3 \times 1}{10 \times 8 \times 6 \times 4 \times 2} \times \frac{\pi}{2} \\ & =\frac{3 \pi}{512} \end{aligned}$$

Question21

$\int_{1/2}^2 |\log_{10} x| dx =$

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Options:

A. $\log_{10}\left(\frac{8}{e}\right)$

B. $\frac{1}{2} \log_{10}\left(\frac{8}{e}\right)$

C. $\log_{10}\left(\frac{2}{e}\right)$

D. $\log_e\left(\frac{3}{e}\right)$

Answer: B

Solution:

$$\int_{1/2}^{2}|\log_{10}x|dx=\int_{1/2}^{1}(-\log_{10}x)dx$$
$$+\int_{1}^{2}\log_{10}x dx$$
$$=-\int_{1/2}^{1}(\log_e x\cdot\log_{10}e)dx$$
$$+\int_{1}^{2}(\log_e x\cdot\log_{10}e)dx$$
$$=-\log_{10}e\int_{1/2}^{1}(\log x)dx+\log_{10}e\int_{1}^{2}(\log x)dx$$
$$=-\log_{10}e[x(\log x-1)]_{\frac{1}{2}}^{1}$$
$$+\log_{10}e[x(\log x-1)]_{1}^{2}\quad\text{(using by parts)}$$
$$=-\log_{10}e\left[1(0-1)-\frac{1}{2}\left(\log\frac{1}{2}-1\right)\right]$$
$$+\log_{10}e[2(\log 2-1)-1(0-1)]$$
$$=\log_{10}e\left[\left(1+\frac{1}{2}\log\frac{1}{2}-\frac{1}{2}\right)+(2\log 2-1)\right]$$
$$=(\log_{10}e)\left(-\frac{1}{2}\log 2+2\log 2-\frac{1}{2}\right)$$
$$=\log_{10}e\left(\frac{3}{2}\log 2-\frac{1}{2}\right)$$
$$=(\log_{10}e)\left(\frac{3}{2}\log_e 2-\frac{1}{2}\log_e e\right)$$
$$=\frac{1}{2}\log_{10}e(\log_e 8-\log_e e)$$
$$=\frac{1}{2}(\log_{10}e)\left(\log_e\left(\frac{8}{e}\right)\right)=\frac{1}{2}\log_{10}\left(\frac{8}{e}\right)$$

Question22

$$\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx=$$

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Options:

A. $\sqrt{2}\log(\sqrt{2}+1)$

B. $\frac{1}{\sqrt{2}}\log(\sqrt{2}+1)$

C. $\log(\sqrt{2}+1)$

D. $\frac{1}{\sqrt{2}}\log(\sqrt{2}-1)$

Answer: B

Solution:

$$I=\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x} d x$$

$$I=\int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin \left(\frac{\pi}{2}-x\right)+\cos \left(\frac{\pi}{2}-x\right)} d x$$

$$=\int_0^{\pi/2} \frac{\cos^2 x}{\cos x+\sin x} d x$$

Adding Eqs. (i) and (ii), we get

$$2 I=\int_0^{\pi/2} \frac{1}{\sin x+\cos x} d x$$

$$2 I=\int_0^{\pi/2} \frac{1}{\sqrt{2}\left(\frac{\sin x}{\sqrt{2}}+\frac{\cos x}{\sqrt{2}}\right)} d x$$

$$2 I=\int_0^{\pi/2} \frac{1}{\sqrt{2} \cos \left(x-\frac{\pi}{4}\right)^2} d x$$

$$2 I=\int_0^{\pi/2} \frac{\sec \left(x-\frac{\pi}{4}\right)}{\sqrt{2}} d x$$

$$2 I=\frac{\ln \left|\sec \left(x-\frac{\pi}{4}\right)+\tan \left(x-\frac{\pi}{4}\right)\right|_0^{\frac{\pi}{2}}}{\sqrt{2}}$$

$$I=\frac{1}{2 \sqrt{2}}[\ln (\sqrt{2}+1)-\ln (\sqrt{2}-1)]$$

$$=\frac{1}{2 \sqrt{2}} \ln \left(\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$$

$$I=\frac{1}{\sqrt{2}} \ln (\sqrt{2}+1)$$

Question23

[.] is the greatest integer function, then

$\int_0^{2 \pi}[|\sin x|+|\cos x|] d x=$

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Options:

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. 2π

Answer: D

Solution:

$I=\int_0^{2\pi}[|\sin x|+|\cos x|]dx$

Period of $|\sin x|+|\cos x|$ is $\frac{\pi}{2}$

$\Rightarrow \quad I=4\int_0^{\frac{\pi}{2}}[|\sin x|+|\cos x|]dx$

For $x\in\left(0,\frac{\pi}{2}\right)$, $\sin x>0$ and $\cos x>0$

$\Rightarrow \quad I=4\int_0^{\frac{\pi}{2}}[\sin x+\cos x]dx$

$\Rightarrow \quad [\sin x+\cos x]=1$

$\Rightarrow \quad I=4\int_0^{\frac{\pi}{2}}dx=4x|_0^{\frac{\pi}{2}}=2\pi$

Question24

If f is defined on R such that $f(x)f(-x)=9$, then $\int_{-23}^{23}\frac{dx}{3+f(x)}=$

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Options:

A. $\frac{51}{3}$

B. $\frac{49}{3}$

C. $\frac{46}{3}$

D. $\frac{46}{6}$

Answer: D

Solution:

Given, $f(x) \cdot f(-x) = 9$

$$f(-x) = \frac{9}{f(x)}$$

$$I = \int_{-23}^{23} \frac{dx}{3 + f(x)}$$

$$I = \int_{-23}^{23} \frac{dx}{3 + f(-x)}$$

$$\left\{ \because \int_a^b f(x) dx = \int_a^b f(a + b - x) dx \right\}$$

$$I = \int_{-23}^{23} \frac{dx}{3 + \frac{9}{f(x)}} \quad \text{[from Eq. (i)]}$$

$$I = \frac{1}{3} \int_{-23}^{23} \frac{f(x)}{3 + f(x)} dx$$

Adding Eqs. (ii) and (iii), we get

$$2I = \int_{-23}^{23} \left(\frac{1}{3 + f(x)} + \frac{1}{3} \frac{f(x)}{3 + f(x)} \right) dx$$

$$= \int_{-23}^{23} \frac{3 + f(x)}{3(3 + f(x))} dx = \frac{1}{3} \int_{-23}^{23} dx$$

$$2I = \frac{46}{3}$$

$$\Rightarrow I = \frac{46}{6}$$
